

Code No: 133BD

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B. Tech II Year I Semester Examinations, April/May - 2023

MATHEMATICS - IV

(Common to CE, EEE, ME, ECE, CSE, EIE, IT, MCT, MMT, AE, MIE, CEE, MSNT)

Time: 3 Hours

Max. Marks: 75

- Note:** i) Question paper consists of Part A, Part B.
 ii) Part A is compulsory, which carries 25 marks. In Part A, Answer all questions.
 iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART – A

(25 Marks)

- 1.a) Define analytic function. [2]
- b) Find the limit of the function $\lim_{z \rightarrow 2i} (3x + iy^2)$. [3]
- c) Determine the poles of the integral $\int_C \frac{4-3z}{z(2-z)} dz$. [2]
- d) Explain about isolated singularity and removable singularity. [3]
- e) Write the cross ratio of four points z_1, z_2, z_3, z_4 . [2]
- f) Find u and v values of the bilinear transformation $w = \cosh z$. [3]
- g) Define Fourier integral theorem. [2]
- h) Define the periodic function. [3]
- i) Write down the one-dimensional wave equation. [2]
- j) Explain the method of separation of variables. [3]

PART – B

(50 Marks)

- 2.a) Check the continuity of $f(z) = \begin{cases} \frac{z}{|z|} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$. [5+5]
- b) Discuss the analyticity of the function $f(z) = z\bar{z}$. [5+5]

OR

- 3.a) Show that $u = \frac{1}{2} \log(x^2 + y^2)$ is harmonic and find its harmonic conjugate function.
- b) Using definition of derivative, find $f'(z)$, if it exists for the following function $f(z) = \frac{1+z}{(1-z)}, z \neq 1$. [5+5]

- 4.a) Find the residue of $f(z) = \frac{z^3}{z^2-1}$ at $z = \infty$.
- b) Evaluate $\oint_C \frac{e^{2z}}{(z+i)^4} dz$ where C is the circle $|z| = 3$. [5+5]

OR

- 5.a) Evaluate the complex integral $\oint_C \tan z dz$ where C is the circle $|z| = 2$.
- b) Find the Taylor's series expansion for the function $f(z) = \left(\frac{z}{z+1}\right)^2$ $z_0 = 1$ about the point z_0 . [5+5]

- 6.a) Using contour integration method prove the integral $\int_0^{2\pi} \frac{x^{a-1}}{1+x} dx = \frac{\pi}{\sin\pi a}$.
- b) Find the bilinear transformation such that the point $z=i$ is the only fixed point and the point '1' is mapped as ∞ . [5+5]

OR

- 7.a) Discuss fully the transformation $w = c \cosh z$, where c is a real number. What physical problem can we study with the help of this transformation?
- b) Evaluate $\int_0^{\infty} \frac{\cos 3\theta}{5+4\cos\theta} d\theta$. [5+5]

- 8.a) Evaluate the Fourier series expansion of $f(x) = e^{-ax}$ in $(0, 2\pi)$.
- b) Find Fourier transform of $f(x) = xe^{-x}$, in $0 \leq x \leq \infty$. [5+5]

OR

- 9.a) Find Fourier sine and cosine transform of $f(x) = 2x$ in $0 < x < 4$.
- b) Find Fourier series expansion of $f(x) = \frac{(\pi-x)}{2}$ in $0 < x < 2$. [5+5]

- 10.a) Solve the equation by method of separation of variables $4 \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$ and $u(0, y) = e^{-5y}$.
- b) Explain the possible solutions of one-dimensional heat equation. [5+5]

OR

11. A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially in a position given $y(x, 0) = y_0 \sin^3 \frac{\pi x}{l}$. If it is released from rest position, find the displacement y at any time and any distance from the end $x=0$. [10]

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